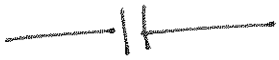
Experiment 4

- Impedance of basic elements

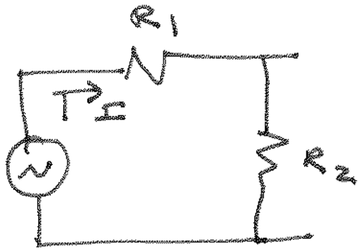
$Z_R = R$

$Z_L = j\omega L$



$Z_c = 1/j\omega C$

- Resistive circuit : current and voltages are all phasor quantities

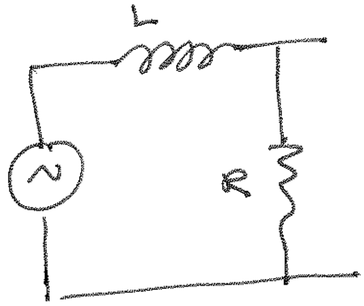


V

$I = \dfrac{V}{R}$

$\Rightarrow$ phase shift = 0 between V and I in a resitive circuit.

- Inductive Circuit



V

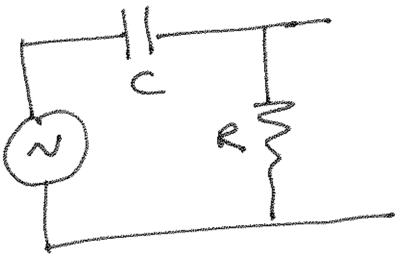
$$I = \frac{V}{Z} = \frac{V}{R + j\omega L}$$

$$= \frac{V}{\sqrt{R^2 + (\omega L)^2}\,\angle \tan^{-1}\frac{\omega L}{R}}$$

$$I = \frac{V}{\sqrt{R^2 + (\omega L)^2}}\,\angle -\tan^{-1}\frac{\omega L}{R}$$

Here, I lags the voltage by $\tan^{-1}\dfrac{\omega L}{R}$

- Capacitive circuit



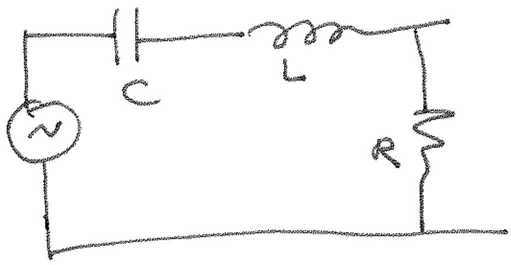
$$I = \frac{V}{Z} = \frac{V}{R + 1/j\omega C}$$

$$= \frac{V}{R - j\frac{1}{\omega C}}$$

$$= \frac{V}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}\,\angle \tan^{-1}\frac{1}{\omega RC}$$

Here, I leads V by $\tan^{-1}\dfrac{1}{\omega RC}$

RLC circuit



$$Z = R + j\omega L + \frac{1}{j\omega C}$$

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\boxed{V = I Z} \quad \text{or} \quad I = \frac{V}{Z}$$

Resonance frequency : frequency at which circuit becomes resistive, i.e.,

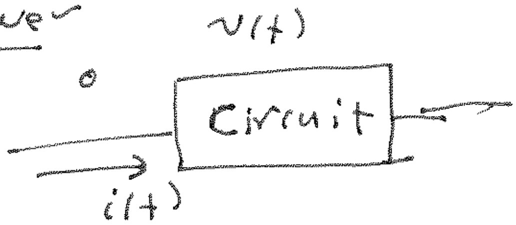
$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^2 L C = 1 \quad \Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad \Rightarrow \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

For $f < f_0$, $|Z_C| > |Z_L| \Rightarrow$ capacitive circuit.

$f > f_0$, $|Z_L| > |Z_C| \Rightarrow$ inductive circuit

Sinusoidal steady-state power



Consider a circuit for which the voltages and current are given by

$$v(t) = V_0 \cos \omega_0 t$$

$$i(t) = I_0 \cos(\omega_0 t + \phi)$$

The instantaneous power is

$$p(t) = V_0 I_0 \cos(\omega_0 t + \phi) \cos \omega_0 t$$

$$p(t) = \frac{V_0 I_0}{2}\left[\cos(2\omega_0 t + \phi) + \cos\phi\right]$$

The average power is

$$P_{av} = \frac{1}{T_0}\int_0^{T_0} p(t)\,dt = \frac{V_0 I_0}{2}\cos\phi$$

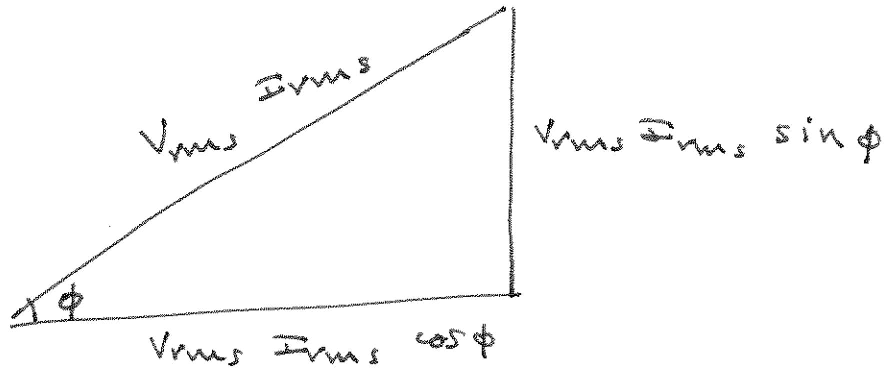
$$P_{av} = \frac{V_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cos\phi = V_{rms} I_{rms} \cos\phi$$

The power Triangle

power factor $= \cos\phi$

active power $= V_{rms} I_{rms} \cos\phi$

rectaive power $= V_{rms} I_{rms} \sin\phi$



RMS Value

consider the voltage waveform

$v(t) = v_0 (\cos \omega t + \phi)$

The rms value of $v(t)$ is

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2 dt} = \frac{v_0}{\sqrt{2}}$$

$$V_{rms} = \frac{\text{peak value}}{\sqrt{2}}$$

The phasor concept

Voltages and currents are represented as complex quantities (no time dependence)

examples: $i(t) = i_0 \cos(\omega t + 30^\circ) \Rightarrow I = \frac{i_0}{\sqrt{2}} \angle \pi/6$

$v(t) = 10 \cos(\omega t) \Rightarrow V = \frac{10}{\sqrt{2}} \angle 0$